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Terceras Jornadas sobre Ecuaciones Diferenciales, Optimización y Análisis Numérico

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INDICE

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Thickness optimization of an elastic beam

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Abstract

An elastic beam of variable thickness subject to a vertical load is considered in this work. Finding the thickness distribution minimizing the compliance of the beam is the structural optimization problem to be solved. Different types of support conditions for the beam are also included in the analysis. The approach is based on the Haslinger and Mäkinen formulation, although the resulting optimization problem is solved in a different way. Finite elements method is used to obtain a suitable discretized problem. The optimization problem is solved by using interior point methods and trust-region strategies. Numerical results are reported.

Resumen

En este trabajo es considerada una viga de espesor variable sujeta a una carga vertical. El problema de optimización estructural a resolver consiste en hallar el espesor que minimiza la deformación de la viga. Se analiza el problema con diferentes tipos de condiciones de soporte en la viga. El trabajo está basado en la formulación propuesta por Haslinger y Mäkinen aunque el problema de optimización obtenido es resuelto con un método diferente. El método de elementos finitos es utilizado para formular el problema discreto. El problema de optimización es resuelto usando el método de puntos interiores y estrategias de región de confianza. Se muestran resultados numéricos.

Key words: nonlinear programming, structural optimization, finite element method.
Palabras Claves: programación no lineal, optimización estructural, método de elementos finitos.
AMS Subject Classification: 74P05, 65L60, 90C30, 90C51.

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1 Introduction

Our main objective in this work is to present an optimization model for solving a structural design problem. Particularly, we are interested in sizing optimization problems, where a typical size of a structure has to be optimized.

The model problem that we considered is well known in the structural optimization literature, however, the modelization technique presented here is simple because it avoids using complex tools and aspects from the sensitivity analysis. The sensitivity analysis is the usual theory for dealing this kind of problem (see [7, 12]). However, the computational aspects and numerical implementation based on it use to be sophisticated (see [1, 5, 8]).

We consider an elastic beam, of length l, subject to a vertical load f responding to the Euler-Bernoulli theory. This theory assumes that cross-sections perpendicular to the axis of the beam remain plane and perpendicular to the axis after deformation. Moreover, the transverse deflection w is given by the following fourth order differential equation

$$\frac{d^2}{dx^2}\left(b\frac{d^2w}{dx^2}\right) = q \quad \text{for} \quad 0 < x < l,$$

where $b = b(x)$ and $q = q(x)$ are given functions of variable x and w is the dependent variable.

Here, b is a function depending on material properties and on the shape of the cross-section area of the beam, q is the distributed load and w represents the transverse deflection. In addition to the differential equation, the deflection w has to satisfy suitable boundary conditions, given by the supporting conditions.

In our optimal design problem the variable e is the thickness of the beam (height of cross-section) and our goal is to find the thickness distribution that minimize the compliance of the beam or, equivalently, that maximize the stiffness of the beam. Under this condition, the cross-section is a function of the thickness, that is, $w = w(e)$.

For numerical experiments we consider a beam with variable thickness, represented by the interval $\Omega = [0, 1]$, and subject to different support conditions.

Though our approach of this problem, and part of its analysis, is based on the formulation of [6], the resulting optimization problem is solved in a different way.

Basically, our idea is to use the finite element method to obtain a suitable discretization of the continuous boundary value problem in order to formulate an optimization nonlinear problem subject to equality, inequality constraints and bounds on the variables. For numerical results we have used the code KNITRO ([15]), which is an implementation of interior point method in combination with trust-region strategies ([14]).

This work is organized as follows. In section 2 we present the thickness beam problem and give a first optimization formulation, in section 3 we develop the discretization of the continuous problem using the finite element method. In section 4 we describe the reformulated optimization problem derived from the discretization and in section 5 we give some numerical results. Finally, in section 6, we state some conclusions.

2 The problem

Let us consider an elastic beam fixed at both of its endpoints subject to a uniformly distributed vertical load $q(x)$ and variable thickness e. Under these assumptions, the deflection w is the solution of the following boundary value problem:

$$\frac{d^2}{dx^2}\left(\beta e(x)^3 \frac{d^2 w}{dx^2}\right) = q(x), \quad 0 < x < l, \tag{2.1}$$
$$w(0) = w(l) = \frac{dw}{dx}(0) = \frac{dw}{dx}(l) = 0,$$

where $\beta \in L^\infty([0,l])$ is a positive function depending on material properties and on the shape of the cross-section area of the beam.

The stiffness of the beam is characterized by the functional $J : H_0^2(\Omega) \to \mathbb{R}$ defined as

$$J(w(e)) = \int_0^l q(x) w(e) dx,$$

where $H_0^2(\Omega)$ represent the Sobolev space of functions whose second derivatives are square-integrable and satisfy the boundary conditions and $w(e)$ is the solution of the boundary value problem (2.1).

This functional, which represents the external energy of deformation, can be considered as a measure of flexibility of the beam. On the other hand, the decrease of this functional implies increasing of the stiffness, so the problem of maximizing the stiffness is equivalent to minimize the functional J. This functional frequently appears in structural optimization either in the objective function or in the constraint set of the problem ([2, 13]).

To formulate the mathematical problem, we add some conditions, besides (2.1), that the thickness e has to satisfy. This conditions define a set of admissible thicknesses C given by:

$$C = \left\{ \begin{array}{l} e \in C^{0,1}(\Omega) \,|\, 0 < e_{\min} \le e \le e_{max} \text{ in } \Omega, \\ |e(x_1) - e(x_2)| \le \gamma |x_1 - x_2| \text{ for all } x_1, x_2 \in \Omega, \\ \int_0^l e(x)dx = \alpha, \quad \text{for some } \alpha > 0, \\ e \text{ is symmetrical in } \Omega \end{array} \right\},$$

i.e., C consists of functions that are uniformly bounded, uniformly Lipschitz continuous in $[0,l]$ and preserve the beam volume and due to the boundary conditions a symmetric property is required. All of such conditions make sense from the practical and physical point of view.

From numerical and implementation aspects we consider the set $\tilde{C}$ characterized by continuous and piecewise constant thickness functions. This last requirement implies to construct a stepped beam. Consequently, the optimization problem is given by

$$\text{Find } e^\star \in \tilde{C} \text{ such that}$$
$$w(e^\star) = \text{argmin } J(w(e)) = \text{argmin} \int_0^l q(x) w(e(x)) dx$$
$$\text{where } w(e) \text{ satisfy the problem (2.1).}$$

To solve numerically this problem, we make a complete discretization of it, and we obtain a new problem defined by a finite number of design parameters.

3 Discretization of the State Problem

For a discretization of the continuous problem we use a finite element approach, that is, we discretize the domain Ω and write the weak formulation of the differential equation. See [11, 10].

Let $d \in \mathbb{N}$ be given and $\Delta h : 0 = a_0 < a_1 < \ldots < a_d = l$ be an equidistant partition of Ω with the step $h = l/d, a_i = ih$, $i = 0, \ldots, d$. Thus, we divide the interval Ω in d subintervals called elements. The $a_i, i = 0 \ldots, d$ are called nodes.

In order to obtain the weak formulation we consider an arbitrary element $\Omega_k = [a_k,\ a_{k+1}]$. Let V_h be a finite dimensional subspace of $H_0^2(\Omega)$, whose functions are continuous piecewise polynomials. Let $v \in V_h$ be an arbitrary test function.

Now, by using the standard integration-by-parts formula twice in Ω_k, we have

$$\int_{a_k}^{a_{k+1}} \left(\beta e^3 \frac{d^2 v}{dx^2} \frac{d^2 w}{dx^2} - vq \right) dx + \left[v \frac{d}{dx} \left(\beta e^3 \frac{d^2 w}{dx^2} \right) - \frac{dv}{dx} \beta e^3 \frac{d^w}{dx^2} \right]_{a_k}^{a_{k+1}} = 0.$$

The variables at each node involve the deflection w and its derivative $\frac{dw}{dx}$.

In the case of a beam supporting a flexion, the terms which have to be evaluated at the endpoints, have structural interpretations ([10]). Thus, the natural boundary conditions involve specifications about the bending moment $\beta e^3 \frac{d^2 w}{dx^2}$ and the shear force $\frac{d}{dx}\left(\beta e^3 \frac{d^2 w}{dx^2}\right)$ at the endpoints.

For simplicity, we introduce the following notation

$$\begin{aligned} Q_1^k &= \left[\tfrac{d}{dx} \left(\beta e^3 \tfrac{d^2 w}{dx^2} \right) \right]_{a_k}, \quad & Q_2^k &= \left[\beta e^3 \tfrac{d^2 w}{dx^2} \right]_{a_k}, \\ Q_3^k &= -\left[\tfrac{d}{dx} \left(\beta e^3 \tfrac{d^2 w}{dx^2} \right) \right]_{a_{k+1}}, \quad & Q_4^k &= -\left[\beta e^3 \tfrac{d^2 w}{dx^2} \right]_{a_{k+1}}. \end{aligned}$$

Thus, the weak formulation is given by

$$\begin{aligned} 0 = & \int_{a_k}^{a_{k+1}} \left(\beta e^3 \frac{d^2 v}{dx^2} \frac{d^2 w}{dx^2} - vq \right) dx - v(a_k) Q_1^k \\ & - \left(-\frac{dv}{dx} \right) Q_2^k - v(a_{k+1}) Q_3^k - \left(-\frac{dv}{dx} \right) Q_4^k. \end{aligned} \tag{3.1}$$

See Fig. 1.

The variational formulation (3.1) requires that the interpolation functions to be twice continuously differentiable and has to satisfy interpolation conditions at the endpoints: $w(a_k),\ w(a_{k+1}),\ w'(a_k),\ w$ These four conditions imply that we need a cubic polynomial to interpolate $w(x)$ and we assign the nodal variables for the element Ω_k

$$\begin{aligned} u_1 &= w(a_k), \quad & u_2 &= -w'(a_k) \\ u_3 &= w(a_{k+1}), \quad & u_4 &= -w'(a_{k+1}). \end{aligned}$$

By using these nodal variables we have the following expression for w in Ω_k

$$\begin{aligned} w(x) &= u_1 \phi_1(x) + u_2 \phi_2(x) + u_3 \phi_3(x) + u_4 \phi_4(x) \\ &= \sum_{j=1}^{4} u_j \phi_j(x), \end{aligned} \tag{3.2}$$

where the functions ϕ_j are the cubic Hermitte polynomials.

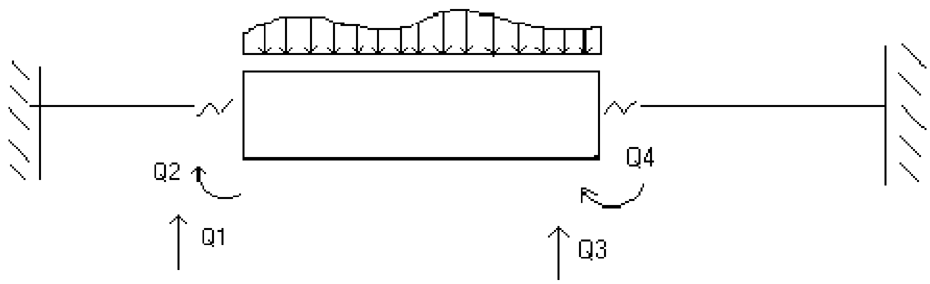

Figure 1: Generalized forces at one element of the beam.

3.1 The Finite Element Model

According to (3.2) and the interpolation functions ϕ_j in the weak formulation (3.1) we obtain the finite element model for the Euler-Bernoulli beam. Since there are four nodal variables at each element, then four possible choices could be used for v: $v = \phi_i, i = 1, \ldots, 4$.

For simplicity we consider the function β with constant value at the whole beam. Therefore, for the *i-th* algebraic equation (for $v = \phi_i$) we have that

$$\begin{aligned} 0 &= \beta e_k^3 \sum_{j=1}^{4} \left(\int_{a_k}^{a_{k+1}} \frac{d^2\phi_j^k}{dx^2} \frac{d^2\phi_i^k}{dx^2} dx \right) u_j \\ &\quad - \int_{a_k}^{a_{k+1}} \phi_i^k q(x) dx - Q_i^k, \end{aligned}$$

or equivalently,

$$\sum_{j=1}^{4} K_{ij}^k u_j^k - F_i^k = 0,$$

where

$$K_{ij}^k = \beta e_k^3 \int_{a_k}^{a_{k+1}} \frac{d^2\phi_j^k}{dx^2} \frac{d^2\phi_i^k}{dx^2} dx,$$

$$F_i^k = \int_{a_k}^{a_{k+1}} \phi_i^k q(x) dx + Q_i^k.$$

Now, if we write these coefficients in matrix notation we have

$$\begin{bmatrix} K_{11}^k & K_{12}^k & K_{13}^k & K_{14}^k \\ K_{21}^k & K_{22}^k & K_{23}^k & K_{24}^k \\ K_{31}^k & K_{32}^k & K_{33}^k & K_{34}^k \\ K_{41}^k & K_{42}^k & K_{43}^k & K_{44}^k \end{bmatrix} \begin{bmatrix} u_1^k \\ u_2^k \\ u_3^k \\ u_4^k \end{bmatrix} = \begin{bmatrix} q_1^k \\ q_2^k \\ q_3^k \\ q_4^k \end{bmatrix} + \begin{bmatrix} Q_1^k \\ Q_2^k \\ Q_3^k \\ Q_4^k \end{bmatrix},$$

where the matrix of the system, which is symmetric, is the stiffness matrix for the k-th element of the beam and the right hand side term is the vector of generalized forces, including the applied external forces and the shear force and flexion at the endpoints of the element.

If h is the length of the element and e is the thickness, the above equation is given by

$$K^k = \frac{2\beta e_k^3}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix},$$

$$F^k = \frac{qh}{12}\left\{\begin{bmatrix} 6 \\ -h \\ 6 \\ h \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix}\right\}.$$

See [11].

3.2 Element Assembly

Having calculated the matrices and equations describing our approximations over each finite element, the next step is to assemble these equations on the entire mesh adding up the contributions furnished by each element. To do this, we take into account the two degrees of freedom at each node. That is, we consider the relationship between the variables associated to the right node at one particular element and the left node to the next element. In particular we consider the equilibrium relationships between the bending moment and the shear force between elements.

When the beam is partioned in d elements we obtain a $2(d+1)\times 2(d+1)$ assembly matrix. However, the algebraic system has $4(d+1)$ unknown variables, $2(d+1)$ correspond to the generalized forces vector and the other $2(d+1)$ to the deformations and its derivatives. Imposing the boundary conditions and the applied loads will allow us to reduce the unknown variables.

The boundary conditions for the beam problem depend on the geometric nature of the support conditions.

In the particular case of a beam fixed at both endpoints the deflection w and its derivative $\frac{dw}{dx}$ are zero at the endpoints. The equilibrium conditions, at the intermediate nodes, yield the equations

$$Q_3^k + Q_1^{k+1} = 0, \qquad Q_4^k + Q_2^{k+1} = 0,$$

because there are no forces applied there, whereas the shear force and the bending moment are unknown at the endpoints of the beam.

Now, by using the equilibrium conditions at the intermediate nodes we have the nonlinear system

$$K(e)U(e) = F(e),$$

where $K(e)$ is the stiffness matrix

$$\begin{bmatrix} 6e_1^3 & -3he_1^3 & \dots & 0 & 0 & 0 \\ -3he_1^3 & 2h^2e_1^3 & \dots & 0 & 0 & 0 \\ -6e_1^3 & 3he_1^3 & 6(e_1^3+e_2^3) & \dots & 0 & 0 \\ -3he_1^3 & h^2e_1^3 & 3h(e_1^3-e_2^3) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 3h(e_{d-1}^3-e_d^3) & -6e_d^3 & -3he_d^3 \\ 0 & \dots & \dots & 2h^2(e_d^3+e_{d-1}^3) & 3he_d^3 & h^2e_d^3 \\ 0 & \dots & \dots & 3he_d^3 & 6e_d^3 & 3he_d^3 \\ 0 & \dots & \dots & h^2e_d^3 & 3he_d^3 & 2h^2e_d^3 \end{bmatrix}$$

and

$$U(e) = \begin{bmatrix} 0 \\ 0 \\ u_3 \\ \vdots \\ \vdots \\ u_{2d} \\ 0 \\ 0 \end{bmatrix}, \qquad F(e) = \frac{qh^4}{24\beta} \left\{ \begin{bmatrix} 6 \\ -h \\ 12 \\ \vdots \\ \vdots \\ 6 \\ h \end{bmatrix} + \begin{bmatrix} Q_1^1 \\ Q_2^1 \\ 0 \\ \vdots \\ 0 \\ Q_3^d \\ Q_4^d \end{bmatrix} \right\}.$$

Since the equations that contain deflections and its slopes do not contain the unknown coefficients associated to the generalized forces, the corresponding equations can be solved independently. Thus, the matrix equation can be partitioned

$$\begin{bmatrix} K^{11} & K^{12} & K^{13} \\ K^{21} & K^{22} & K^{23} \\ K^{31} & K^{32} & K^{33} \end{bmatrix} \begin{bmatrix} U^1 \\ U^2 \\ U^3 \end{bmatrix} = \begin{bmatrix} F^1 \\ F^2 \\ F^3 \end{bmatrix},$$

where U^1 and U^3 contain the known deflection and derivatives (in this case are zero due the support conditions) and U^2 are the unknown. At the right hand side, in F^1 and F^3 there are unknown coefficients. Under this conditions the matrix equation that we are interested in, can be expressed as

$$K^{22}U^2 = F^2. \tag{3.3}$$

This is a $2(d-1)\times 2(d-1)$ system, whose unknown variables are the deflection and derivatives at the intermediate nodes of the discretization. The coefficients of the submatrix K^{22} depend on the thickness of each element of the beam.

4 Reformulation of the Optimization Problem

The discretization of the state problem yields a system of nonlinear algebraic equations (3.3). For the sake of simplicity of notation we rewrite the system (3.3) as

$$K(e)U(e) = F, \tag{4.1}$$

where $U(e)$ contains the deflection and its derivatives at the intermediate nodes. For the numerical solution of the problem we have to consider the conditions for the admissible thickness and discretization of the objective function.

The discretization of the state problem concern to the functional J, however, the restriction of J to the subspace V_h is identified with another functional defined in the euclidean space $\mathbb{R}^{2(d-1)}$ as in [6].

To do this, we discretize the integral by using the same numerical formula that we have used for the volume of the beam. Since we consider uniformly distributed load, that is $q(x) = q$, constant at the whole beam, and we also assume that the thickness is constant at each element it is enough to use the rectangle rule. Clearly, if the load were not uniformly distributed it would be better to use a higher order quadrature rule. Then, the objective function is approximated by

$$J(u(e)) = \int_0^l q(x)u(e(x))dx \simeq q^T\bar{U},$$

where $\bar{U}$ contains only the $d-1$ components of $U(e)$ associated to deflections at the intermediate nodes and q is a constant vector, which has the same size.

Since the discretized beam is stepped constant, the admissible thickness is defined by a piecewise constant function that satisfies the following conditions

$$e_{min} \leq e_i \leq e_{max}, \quad i = 1, \ldots, d,$$

$$| e_{i+1} - e_i | \leq \gamma h, \quad i = 1, \ldots, d-1,$$

for some constant γ. Moreover, the preserving volume condition is given by

$$h \sum_{i=1}^{i=d} e_i = \alpha.$$

Finally, the optimization problem is given by

$$\begin{array}{ll} \min\limits_{\mathrm{e,U}} & q^T \bar{U} \\ \text{s.t.} & K(e)U(e) = F \\ & h \sum\limits_{i=1}^{i=d} e_i = \alpha, \\ & e_{min} \leq e_i \leq e_{max}, \quad i = 1, \ldots, d, \\ & | e_{i+1} - e_i | \leq \gamma h, \quad i = 1, \ldots, d-1. \end{array}$$

Thus, the above problem is the classical formulation of a nonlinear programming problem with bounds at the variables and equality and linear inequality constraints. It is worth noting that system of algebraic equations (4.1) is nonlinear strongly on the design variables e_i. Through the derivation of the model is similar to the presentation in [6], the formulation is conceptually different. In our approach the variables corresponding to thickness and deflection are related through the constraint equation $KU = F$, although initially they are independent. Moreover, in that presentation the optimization problem take into account the thicknesses and their relationship with the deflections trough a sensitivity analysis. While sensitivity analysis provides an elegant and mathematically rigorous method to formulate the problem, it has one very serious drawback: the numerical solution is very difficult to be computed.

5 Numerical Experiments

For numerical experiments we consider a beam with two different support conditions. First, we analyze a beam with both endpoints fixed, whose discretized model has been described in previous sections. Then, we consider a beam with one endpoint fixed and the other one simply supported. For this case, we note that the boundary conditions are different. At the simply supported endpoint the deflection is zero whereas its derivative is not: this support condition does not allow to assimilate bending moment at this endpoint. By using the notation of section 3, we have

$$w(0) = u_1^1 = 0; \; \frac{dw}{dx}\bigg|_{x=0} = u_2^1 = 0;$$

$$w(l) = u_3^d = 0;\ Q_4^d = -\left[\beta e_d^3 \frac{d^2 w}{dx^2}\right]_l = 0.$$

According to these boundary conditions the nonlinear system of algebraic equations from the finite element method is modified.

For solving the optimization problem, we have used the solver KNITRO 3.1 ([14, 15]), by means of a Visual Fortran interface. The user has to provide three subroutines including the data of the problem, the objective function and the constraints, and finally, the gradient of the objective function and the Jacobian matrix of the constraints. This code is an implementation of the interior point method, where the nonlinear programming problem is solved by means of the solution of a sequence of barrier subproblems, depending on a parameter μ. On the other hand, the algorithm uses a trust-region method as a globalization strategy and a merit function to obtain global convergence results.

Each iteration of the subalgorithm generates steps whose normal and tangential components satisfy mild conditions on adequate models. The normal component improves the feasibility and the tangential component, which is computed using the projected conjugated gradient method, improves the optimality. The whole code KNITRO 3.1 computes some iterations for each subproblem before the barrier parameter is decreased and then the procedure is repeated until suitable convergence conditions are reached. See [4, 3] for a complete description and analysis of this method.

The number of elements used for numerical experiments in both problems was $d = 8$ and $d = 32$. For simplicity we adopted the following default values:

- $l = 1$, the length of the beam.
- $\beta = 1; q(x) = -1$, the uniformly distributed load.
- $e_{\min} = 0.01$, $e_{\max} = 0.1$, bounds for the thickness.
- $\alpha = 0.05$, $\gamma = 0.5$.

The initial approximation for thickness were $e_i = 0.05, i = 1, \ldots, d$. Then we solved the linear system of algebraic equations (4.1) in order to obtain the initial values for the other variables.

Finally, we show in Fig. (2–3), the profile of the optimal solution for the beam problem, in both cases, and the corresponding optimal value of the objective function.

6 Conclusions

We have presented and tested an optimization formulation for the elastic beam problem, which is subject to vertical loads, as in Euler-Bernoulli theory.

One of the main advantage of this formulation is that it is possible to use any of the many available nonlinear programming method to solve the thickness problem. It is a very attractive feature due to the advances in optimization algorithm during the last years. On the other hand, researchers in engineering, applied mathematics and other sciences paid attention to the conexion between optimization and computational mechanics in structural problems (structural optimization), so important and interesting advances were done about this area recently.

Though we considered here a one dimensional model, the same ideas can be applied in more complicated structural problems of dimension 2 or 3, or subject to different support and loads.

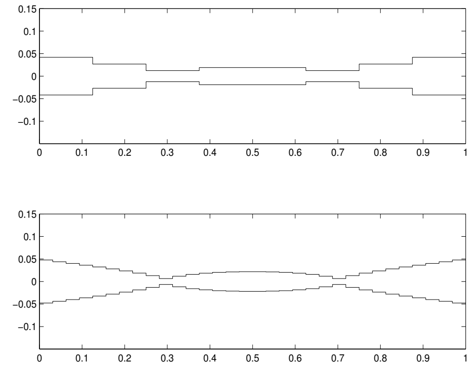

Figure 2: Discretization of the beam with both endpoints fixed by using 8 (J=6.388) and 32 elements (J=5.659) respectively.

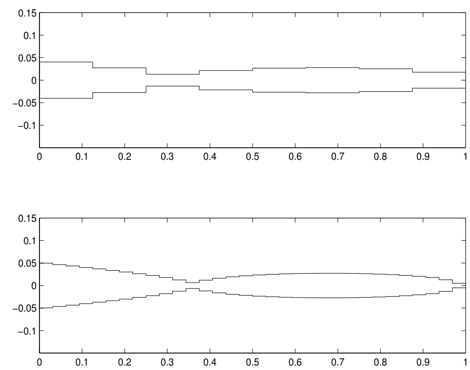

Figure 3: Discretization of the beam with endpoints fixed-simply supported by using 8 (J=29.458) and 32 elements (J=15.765) respectively.

The numerical results obtained are promising and encouraging. Moreover, the particular structure of the elastic beam problem obtained after the discretization seems to indicate that new optimization approach as Inexact–restoration methods ([9]) could be perform efficiently, particularly for large scale discretization.

In practice, it is usually important to optimize structures subject to different types of loads. In addition to the compliance cost functional we could consider another two functionals involving the smallest eigenvalues for two generalized problems. Eigenvalues represent natural frequencies of free oscillations and buckling loads of the beam and depend on the thickness distribution e. The goal could be to find a thickness minimizing the compliance of the perpendicularly loaded beam, maximizing the minimal natural frequency (i.e., the beam is stiffer under slowly varying dynamic forces), and maximizing the minimal buckling load (i.e., the beam does not loose its stability easily under the compressive load). So we would have a simple prototype of multiobjective thickness optimization of an elastic beam. It could be solved using nonlinear least squares if the solution of each problem is known. This will be the object of our practical research in the near future.

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