MATHEMATICAL CONSIDERATIONS ON THE HEAT TRANSFER WITH PHASE CHANGE WITH NEGLIGIBLE LATENT HEAT

DOMINGO A. TARZIA*, *, LUIS T. VILLA**, *****

**PROMAR (CONICET-UNR), Instituto de Matemática "Beppo Levi", Facultad de Ciencias Exactas e Ingeniería, Universidad Nacional de Rosario, Av. Pellegrini 250, (2000) Rosario, Argentina.*
***INIQUI (CONICET-UNSa), Facultad de Ciencias Tecnológicas, Universidad Nacional de Salta, Buenos Aires 177, (4400) Salta, Argentina.*
****Investigador del Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina.*

Abstract

We shall prove that the Neumann's mathematical model of free boundary for a one-dimensional phase-change process with negligible latent heat can be understood as a limit to the respective model of the same process when latent heat tends to zero.

Firstly, we shall consider convergence in the case in which the phase-change occurs without mass density jump and secondly, we shall generalize the above study to the case in which the convection in the liquid phase is induced by the density jump at the solid-liquid interface.

If the temperature in i-phase ($i = 1$: solid phase, $i = 2$: liquid phase) with latent heat $\ell > 0$ (with negligible latent heat) is called $T_i = T_i(x, t, \ell)$ ($T_i = T_i(x, t)$) and the free boundary for $\ell > 0$ (for negligible ℓ) is called $s_\ell = s_\ell(t)$ ($s_0 = s_0(t)$), then we obtain the following estimates (valid for the two cases):

$$0 < \frac{s_o(t) - s_\ell(t)}{\sqrt{t}} \leqslant C_1\, \ell, \qquad \text{(i)}$$

$$|T_2(x, t) - T_2(x, t, \ell)| \leqslant C_2\, \ell, \qquad \text{(ii)}$$

$$|T_1(x, t) - T_1(x, t, \ell)| \leqslant C_3\, \ell \qquad \text{(iii)}$$

where C_1, C_2 and C_3 are appropiate constants independt of ℓ.

Introduction

The present paper was motivated by the recent one by Guzmán (1982), who considered a one-dimensional solidification case with negligible latent heat. It we bear in mind that Sherman (1971), studied convergence when $\ell \to 0$ for a one-dimensional one-phase Stefan problem and that Tarzia (1979, 1983) studied convergence when $\ell \to 0$ in the variational inequality corresponding to the multidimensional two-phase Stefan problem, this paper justifies, from a mathematical point of view, the hypotheses carried out by Guzmán (1982), by using Newmann's solution.

In the first part, we shall consider convergence in the case in which the phase-change occurs without mass density jump in a two phase one-dimensional fusion problem, and in the second part, such study will be generalized to the case in which the convection in the liquid phase is induced by the mass density jump at the solid-liquid interface through a two-phase one-dimensional solidification. Closed formulas for the two-phase Stefan problem can be found in Bancora-Tarzia (1985) and Carslaw-Jaeger (1959).

I. Phase-change with negligible jump mass density

We shall consider the case of fusion of a two-phase Stefan problem for a one-dimensional semi-infinite material with constant thermal properties and negligible jump mass density, that is $\rho_1 = \rho_2$.

Without loss of generality, we shall take null the phase-change temperature. We shall find the function $s_\ell = s_\ell(t) > 0$ (free boundary), defined for $t > 0$, and the temperature

$$T(x,t,\ell) = \begin{cases} T_2(x,t,\ell) > 0 & \text{if} \quad 0 < x < s_\ell(t),\ t>0 \\ 0 & \text{if} \quad x = s_\ell(t),\ t>0 \\ T_1(x,t,\ell) < 0 & \text{if} \quad x > s_\ell(t),\ t>0 \end{cases} \quad \text{(I-1)}$$

defined for $x > 0$, $t > 0$, so that they satisfy the following conditions:

$$a_1^2 T_{1xx} - T_{1t} = 0 \ , \quad x > s_\ell(t) \ , \quad t>0 \quad \text{(I-2)}$$

$$a_2^2 T_{2xx} - T_{2t} = 0 \ , \quad 0 < x < s_e(t), \ t>0 \quad \text{(I-3)}$$

$$T_1(s_\ell(t), t, \ell) = T_2(s_\ell(t), t, \ell) = 0, \ t>0 \quad \text{(I-4)}$$

$$k_1 T_{1x}(s_\ell(t), t, \ell) - k_2 T_{2x}(s_\ell(t), t, \ell) = \rho \ell \dot{s}_\ell(t) \ , \quad t>0 \quad \text{(I-5)}$$

$$T_1(x, 0, \ell) = T_1(\infty, t, \ell) = -C < 0 \ , \ x>0 \ , \ t>0 \quad \text{(I-6)}$$

$$T_2(0, t, \ell) = B > 0 \ , \quad t>0 \quad \text{(I-7)}$$

$$s_\ell(0) = 0 \ , \quad \text{(I-8)}$$

where ρ is the common mass density to both phases.

The solution of problem (I-2)-(I-8) is given by:

$$\begin{cases} T_1(x,t,\ell) = -C + \dfrac{C}{F\left(\dfrac{\sigma_\ell}{a_1}\right)} F\left(\dfrac{x}{2a_1\sqrt{t}}\right) \\ T_2(x,t,\ell) = B - \dfrac{B}{f\left(\dfrac{\sigma_\ell}{a_2}\right)} f\left(\dfrac{x}{2a_2\sqrt{t}}\right) \\ s_\ell(t) = 2\sigma_\ell\sqrt{t} \ , \ \sigma_\ell > 0 \ , \end{cases} \quad \text{(I-9)}$$

where σ_ℓ is the unique solution of the equation

$$G(x) = \ell x \ , \ x>0 \ ; \quad \text{(I-10)}$$

with

$$\begin{cases} G(x) = \dfrac{Bk_2}{\rho a_2\sqrt{\pi}} F_2\left(\dfrac{x}{a_2}\right) - \dfrac{Ck_1}{\rho a_1\sqrt{\pi}} F_1\left(\dfrac{x}{a_1}\right) \\ F_1(x) = \dfrac{\exp(-x^2)}{1-f(x)}; \quad F_2(x) = \dfrac{\exp(-x^2)}{f(x)} \\ f(x) = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_0^x \exp(-u^2)\,du \ (\equiv \operatorname{erf}(x)) \ , \\ F(x) = 1 - f(x) \ (\equiv \operatorname{erfc}(x)). \end{cases} \quad \text{(I-11)}$$

Function G satisfies the following properties:

$$G(0^+) = +\infty \ , \quad G(+\infty) = -\infty \ , \quad G' < 0 . \quad \text{(I-12)}$$

Moreover, the solution of problem (I-2)-(I-8) with negligible latent heat, that is, $\ell = 0$, is given by:

$$\begin{cases} T_1(x,t) = -C + \dfrac{C}{F\left(\dfrac{\sigma_0}{a_1}\right)} F\left(\dfrac{x}{2a_1\sqrt{t}}\right) \\ T_2(x,t) = B - \dfrac{B}{f\left(\dfrac{\sigma_0}{a_2}\right)} f\left(\dfrac{x}{2a_2\sqrt{t}}\right) \\ s_0(t) = 2\sigma_0\sqrt{t} \ , \quad \sigma_0 > 0 \end{cases} \quad \text{(I-13)}$$

where σ_0 is the unique solution of the equation

$$G(x) = 0 \ , \quad x > 0 . \quad \text{(I-14)}$$

Property I-1

Function $\sigma_\ell = \sigma(\ell) > 0$, defined in $(0, +\infty)$, satisfies the following properties:

i) $\sigma_\ell = \sigma(\ell)$ is a decreasing function with ℓ which verifies:

$$\sigma(0^+) = \sigma_0 \ , \quad \sigma(+\infty) = 0 \quad \text{(I-15)}$$

ii) There exists a constant $\alpha_1 > 0$ (independent of ℓ), so that we have:

$$0 < \sigma_0 - \sigma_\ell \leqslant \alpha_1 \ell \ , \ \forall \ \ell \in (0, \ell_0) \ ,$$

$$\ell_0 = \text{const.} > 0 \ , \quad \text{(I-16)}$$

that is, $\lim_{\ell \to 0} \sigma_\ell = \sigma_0$

Proof: i) arises from properties (I-12).

ii) Using elementary considerations, we have

$$\frac{\ell\,\sigma_\ell}{\sigma_0-\sigma_\ell} = \tan(\pi-\beta) = -\tan\beta = -G'(\xi) \quad \text{(I-17)}$$

for some $\xi \in (\sigma_\ell, \xi_0)$ (see Fig. 1).

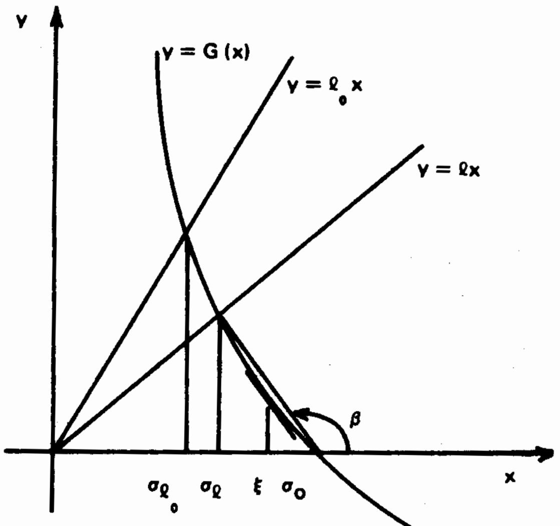

Fig. 1

From (I-17) we deduce

$$0<\sigma_0-\sigma_\ell = \frac{-1}{G'(\xi)}\,\ell\,\sigma_\ell \leqslant \alpha_0\,\sigma_0\,\ell = \alpha_1\,\ell, \quad \text{(I-18)}$$

where

$$\begin{cases} \alpha_0 = \underset{\xi\in[\sigma_{\ell_0},\sigma_0]}{\text{máx}} \dfrac{1}{|G'(\xi)|} = \\ \alpha_0\,(\ell_0, \sigma_0, \sigma_{\ell_0}, B, k_2, \rho, a_2, C, k_1, a_1), \\ \alpha_1 = \alpha_0\,\sigma_0\,, \end{cases} \quad \text{(I-19)}$$

where we have considered $\ell \in (0, \ell_0)$ for some given data $\ell_0 > 0$, because we are only interested in taking $\ell \to 0$.

Remark I-1

Taking into account the expressions of $s_0\,(t)$ and $s_\ell\,(t)$, given by (I-13) and (I-9), respectively, we deduce the following property:

$$0<\frac{s_0\,(t)-s_\ell\,(t)}{\sqrt{t}} = 2\,(\sigma_0-\sigma_\ell) \leqslant \alpha_2\,\ell\,, \quad \text{(I-20)}$$

where $\alpha_2 = 2\,\alpha_1$ is a constant independent of ℓ.

Property I-2

The temperatures of the solid and liquid phases concerning cases $\ell > 0$ and $\ell = 0$ satisfy the following inequalities:

$$\text{i) } 0<T_2\,(x,t)-T_2\,(x,t,\ell) \leqslant \alpha_3\,\ell\,,$$

$$0<x<s_\ell\,(t)\,,\quad t>0 \quad \text{(I-21)}$$

$$\text{ii) } 0<T_1\,(x,t)-T_1\,(x,t,\ell) \leqslant \alpha_4\,\ell\,,$$

$$x>s_0\,(t)\,,\quad t>0\,, \quad \text{(I-22)}$$

where α_3 and α_4 are two positive constants independent of ℓ.

Proof: 1) Taking into account the expressiones given by (I-9) and (I-13), the monotony of function f and the inequality (I-16), we deduce that:

$$0<T_2\,(x,t)-T_2\,(x,t,\ell) =$$

$$Bf\left(\frac{x}{2\,a_2\sqrt{t}}\right)\frac{f\left(\frac{\sigma_0}{a_2}\right)-f\left(\frac{\sigma_\ell}{a_2}\right)}{f\left(\frac{\sigma_\ell}{a_2}\right)f\left(\frac{\sigma_0}{a_2}\right)} \leqslant$$

$$B\frac{2}{\sqrt{\pi}}\,\frac{\sigma_0-\sigma_\ell}{a_2}\,\exp\left(-\frac{\sigma_\ell^2}{a_2^2}\right) \leqslant \alpha_3\,\ell\,,$$

where

$$\alpha_3 = \frac{2\,B\,\alpha_1}{a_2\sqrt{\pi}}\,\frac{F_2\left(\frac{\sigma_{\ell_0}}{a_2}\right)}{f\left(\frac{\sigma_0}{a_2}\right)} \quad \text{(I-23)}$$

2) Similarly to what was done in 1), we deduce

$$0<T_1\,(x,t)-T_1\,(x,t,\ell) =$$

$$= C\,F\left(\frac{x}{2a_1\sqrt{t}}\right)\frac{f\left(\frac{\sigma_0}{a_1}\right)-f\left(\frac{\sigma_\ell}{a_1}\right)}{F\left(\frac{\sigma_0}{a_1}\right)F\left(\frac{\sigma_\ell}{a_1}\right)} \leqslant$$

$$\leqslant \frac{2\,C}{a_1\sqrt{\pi}}\,\frac{\exp(-\sigma_\ell^2/a_1^2)(\sigma_0-\sigma_\ell}{F\left(\frac{\sigma_\ell}{a_1}\right)F\left(\frac{\sigma_0}{a_1}\right)} \leqslant \alpha_4\,\ell$$

where

$$\alpha_4 = \frac{2 C \alpha_1}{a_1 \sqrt{\pi}} \frac{F_1\left(\frac{\sigma_0}{a_1}\right)}{F\left(\frac{\sigma_0}{a_1}\right)} \tag{I-24}$$

Remark I-2

The convergence obtained in 1) and 2) for the Property I-2 is uniform in variables x and t in all the closed sets included in the domain, given by (I-21) and (I-22), respectively.

Remark I-3

From Property 2 it is clear that the model considered by Guzmán (1982), may be obtained as the limit of Neumann's model, for a two-phase one-dimensional Stefan problem when latent heat tends to zero.

Next, we shall generalize the previous properties to the case in which a solidification problem for a semi-infinite material with a mass-density jump at the free boundary, that is $\rho_1 \neq \rho_2$ is considered.

II. Phase change with density jump at the free boundary

If we consider a process of solidification of a semi-infinite material which takes into account the density jump, that is $\rho_1 \neq \rho_2$, at the free boundary, then the problem could be formulated through the following two-phase Stefan problem: We shall find function $s_\ell = s_\ell(t) > 0$ (free boundary), defined for $t > 0$, and the temperature

$$T(x,t,\ell) = \begin{cases} T_1(x,t,\ell) < 0 & \text{if } 0 < x < s_\ell(t),\ t>0 \\ 0 & \text{if } x = s_\ell(t),\ t>0 \\ T_2(x,t,\ell) > 0 & \text{if } x > s_\ell(t),\ t>0 \end{cases} \tag{II-1}$$

defined for $x > 0$ and $t > 0$, so that they satisfy the following conditions:

$$a_1^2 T_{1xx} - T_{1t} = 0, \quad 0 < x < s(t), \quad t > 0 \tag{II-2}$$

$$a_2^2 T_{2xx} + \frac{\rho_1 - \rho_2}{\rho_2} \dot{s}_\ell(t)\, T_{2x} - T_{2t} = 0,$$

$$x > s_\ell(t), \quad t > 0 \tag{II-3}$$

$$T_1(s_\ell(t), t, \ell) = T_2(s_\ell(t), t, \ell) = 0, \quad t > 0 \tag{II-4}$$

$$k_1 T_{1x}(s_\ell(t), t, \ell) - k_2 T_{2x}(s_\ell(t), t, \ell) = \rho_1 \ell \dot{s}_\ell(t), \quad t > 0 \tag{II-5}$$

$$T_2(x, 0, \ell) = T_2(+\infty, t, \ell) = E > 0, \quad x > 0, \quad t > 0 \tag{II-6}$$

$$T_1(0, t, \ell) = -D < 0, \quad t > 0 \tag{II-7}$$

$$s_\ell(0) = 0, \tag{II-8}$$

The solution of problem (II-2)-(II-8) is given by Bancora-Tarzia, 1985:

$$\begin{cases} T_1(x,t,\ell) = -D + \dfrac{D}{f\left(\frac{\gamma_\ell}{a_1}\right)} f\left(\dfrac{x}{2 a_1 \sqrt{t}}\right) \\ T_2(x,t,\ell) = \dfrac{E}{F\left(\frac{\gamma_\ell}{a_0}\right)} \cdot \left[f\left(\delta_\ell + \dfrac{x}{2 a_2 \sqrt{t}}\right) - f\left(\dfrac{\gamma_\ell}{a_0}\right)\right] \\ s_\ell(t) = 2 \gamma_\ell \sqrt{t}, \quad \gamma_\ell > 0, \end{cases} \tag{II-9}$$

where

$$\varepsilon = \frac{\rho_1 - \rho_2}{\rho_2}, \quad \delta_\ell = \frac{\gamma_\ell |\varepsilon|}{a_2}; \quad a_0 = \frac{a_2}{1 + |\varepsilon|}, \tag{II-10}$$

and γ_ℓ is the unique solution of the equation

$$H(x) = \ell x, \quad x > 0, \tag{II-11}$$

with

$$H(x) = \frac{k_1 D}{\rho_1 a_1 \sqrt{\pi}} F_2\left(\frac{x}{a_1}\right) - \frac{k_2 E}{\rho_1 a_2 \sqrt{\pi}} F_1\left(\frac{x}{a_0}\right) \tag{II-12}$$

that satisfies the following properties:

$$H(0^+) = +\infty\,,\quad H(+\infty) = -\infty\,,\quad H' < 0\,. \qquad \text{(II-13)}$$

Moreover, the solution of problem (II-2)-(II-8) with negligible latent heat, that is $\ell = 0$, is given by:

$$\begin{cases} T_1(x,t) = -D + \dfrac{D}{f\left(\dfrac{\gamma_0}{a_1}\right)}\, f\left(\dfrac{x}{2a_1\sqrt{t}}\right) \\[2ex] T_2(x,t) = \dfrac{E}{F\left(\dfrac{\gamma_0}{a_0}\right)} \cdot \left[f\left(\delta_0 + \dfrac{x}{2a_2\sqrt{t}}\right) - f\left(\dfrac{\gamma_0}{a_0}\right)\right] \\[2ex] s_0(t) = 2\gamma_0\sqrt{t}\,,\quad \gamma_0 > 0\,,\quad \delta_0 = \dfrac{\gamma_0\,|\varepsilon|}{a_2}\,, \end{cases} \qquad \text{(II-14)}$$

where γ_0 is the unique solution of the equation

$$H(x) = 0\,,\quad x > 0\,. \qquad \text{(II-15)}$$

Property II-1

A) Funciton $\gamma_\ell = \gamma(\ell) > 0$, defined in $(0, +\infty)$, satisfies the following properties:

i) $\gamma_\ell = \gamma(\ell)$ is a decreasing function with ℓ that verifies:

$$\gamma(0^+) = \gamma_0\,,\quad \gamma(+\infty) = 0 \qquad \text{(II-16)}$$

ii) There exists a constant $A_1 > 0$ (independent of ℓ) so that we have

$$0 < \gamma_0 - \gamma_\ell \leq A_1\,\ell\,, \qquad \text{(II-17)}$$

that is, $\lim_{\ell \to 0} \gamma_\ell = \gamma_0$

iii) There exists a constant $A_2 > 0$ (independent of ℓ), so that the free boundaries s_0 and s_ℓ are related through the following expresion

$$0 < \frac{s_0(t) - s_\ell(t)}{\sqrt{t}} \leq A_2\,\ell\,. \qquad \text{(II-18)}$$

B) The temperatures of the solid and liquid phases concerning cases $\ell > 0$ and $\ell = 0$ satisfy the following inequalities:

i) $0 < T_1(x,t,\ell) - T_1(x,t) \leq A_3\,\ell\,,$

$$0 < x < s_\ell(t)\,,\quad t > 0 \qquad \text{(II-19)}$$

ii) $|T_2(x,t,\ell) - T_2(x,t)| \leq A_4\,\ell\,,$

$$x > s_0(t)\,,\quad t > 0 \qquad \text{(II-20)}$$

where A_3 and A_4 are two positive constants independent of ℓ.

Proof

(A i, ii, iii) and (B i) are analogous to what was done in part 1 and we have

$$A_1 = \beta_0\,\gamma_0\,,\quad A_2 = 2A_1 \qquad \text{(II-21)}$$

$$\beta_0 = \underset{\xi \in [\gamma_{\ell_0}, \gamma_0]}{\text{Máx}} \frac{1}{|H'(\xi)|} =$$

$$\beta_0\left(\ell_0, \gamma_0, \gamma_{\ell_0}, D, E, k_2, k_1, a_1, \rho_1, a_2, a_0\right)$$

$$A_3 = \frac{2DA_1\,F_2(\gamma_{\ell_0}/a_1)}{a_1\sqrt{\pi}\;\; f\left(\dfrac{\gamma_0}{a_1}\right)} \qquad \text{(II-22)}$$

(B ii) Using elementary considerations, we have

$$\frac{T_2(x,t,\ell) - T_2(x,t)}{E} =$$

$$= \frac{F\left(\dfrac{\gamma_\ell}{a_0}\right) - F\left(\delta_\ell + \dfrac{x}{2a_2\sqrt{t}}\right)}{F\left(\dfrac{\gamma_\ell}{\epsilon_0}\right)} - \frac{F\left(\dfrac{\gamma_0}{a_0}\right) - F\left(\delta_0 + \dfrac{x}{2a_2\sqrt{t}}\right)}{F\left(\dfrac{\gamma_0}{a_0}\right)} =$$

$$= \frac{F\left(\delta_0 + \dfrac{x}{2a_2\sqrt{t}}\right) - F\left(\delta_\ell + \dfrac{x}{2a_2\sqrt{t}}\right)}{F\left(\dfrac{\gamma_\ell}{a_0}\right)} +$$

$$+ \frac{F\left(\delta_0 + \dfrac{x}{2a_2\sqrt{t}}\right)}{F\left(\dfrac{\gamma_\ell}{a_0}\right)\,F\left(\dfrac{\gamma_0}{a_0}\right)} \left[F\left(\frac{\gamma_\ell}{a_0}\right) - F\left(\frac{\gamma_0}{a_0}\right)\right]$$

and then we obtain

$$|T_2(x,t,\ell) - T_2(x,t)| \leq$$

$$\leq \frac{2E}{\sqrt{\pi}\, F\left(\frac{\gamma_0}{a_0}\right)}\left[\frac{|\varepsilon|}{a_2}(\gamma_0 - \gamma_\ell) + \right.$$

$$\left. + \frac{\exp(-\gamma_{\ell_0}^2 / a_0^2)\,(\gamma_0 - \gamma_\ell)}{F\left(\frac{\gamma_0}{a_0}\right) \qquad a_0}\right]$$

that is (II-20) with

$$A_4 = \frac{2EA_1}{a_2\sqrt{\pi}\, F\left(\frac{\gamma_0}{a_0}\right)}$$

$$\left[|\varepsilon| + \frac{(1+|\varepsilon|)}{F\left(\frac{\gamma_0}{a_0}\right)}\exp(-\gamma_{\ell_0}^2 / a_0^2)\right] \qquad \text{(II-23)}$$

Acknowledgment

This paper has been partially sponsored by the Project "Problemas de Fronteras Libres de la Física-Matemática".

Nomenclature

$a^2 = \frac{k}{\rho c}$	thermal diffusivity
$B > 0$	temperature in the fixed face $x = 0$
$-C < 0$	initial temperature
c	specific heat
$-D < 0$	temperature in the fixed face $x = 0$
$E > 0$	initial temperature
F	function defined in (I-11)
f	function defined in (I-11)
F_1	function defined in (I-11)
F_2	function defined in (I-11)
G	function defined in (I-11)
H	function defined in (II-12)
k	thermal conductivity
ℓ	latent heat
$s(t)$	position of the solid-liquid interface (free boundary) at time $t > 0$
T	temperature
$t > 0$	time
$x > 0$	space coordinate variable

Greek letters

γ	coefficient which characterizes the free boundary s in paragraph II
δ	coefficient defined in (II-10)
ε	coefficient defined in (II-10)
σ	coefficient which characterizes the free boundary s in paragraph I
ρ	mass density

Subscripts

1	solid phase
2	liquid phase
ℓ	when we take the latent heat as variable
0	when we take $\ell = 0$

References

Bancora, A.B., D.A. Tarzia, "On the Neumann solution for the two-phase Stefan problem including the density jump at the free boundary", *Lat. Am. J. Heat Mass Transfer*, 9, 215-222 (1985).

Carslaw, H.S., J.C. Jaeger, *Conduction of heat in solids*, Oxford University Press (Clarendon), London (1959).

Guzmán, J.A., "Application of the heat transfer solution for the case of one-dimensional solidification with negligible latent heat", *Lat. Am. J. Heat Mass Transfer*, 6, 41-51 (1982).

Sherman, B., "Limiting behavior in some Stefan problems as the latent heat goes to zero", SIAM *J. Appl. Math.*, **20**, 319-327 (1971).

Tarzia, D.A., "Sur le probleme de Stefan à deux phases", *C.R. Acad. Sc. Paris*, **288A**, 941-944 (1979).

Tarzia, D.A., "Etude de l'inéquation variationnelle proposée par Duvaut pour le probleme de Stefan à deux phases, II", *Boll. Un. Mat. Italiana*, **2B**, 589-603 (1983).

(Received July 7, 1986; accepted for publication December 30, 1987).

Due errors in the transcription of the autors' manuscript; this paper originally published as LAAR, Vol. 20, N° 2, pp. 89-94 is included again in this issue.