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AN ALTERNATIVE FORMULA TO COMPUTE THE NUTRIENT UPTAKE FOR ROOTS

J. C. Reginato[1],* and D. A. Tarzia[2]

[1]Department of Chemistry–Physics, Sciences Faculty,
National University of Río Cuarto, Ruta 36, Km 601, Rio
Cuarto 5800, Argentina
[2]Department of Mathematics and CONICET, Sciences
Management Faculty, Austral University, Paraguay 1950,
Rosario S2000FZF, Argentina

ABSTRACT

Various authors with small variations have used different expressions in order to compute the nutrient uptake for crop roots. These expressions use the nutrient concentration on the root surface and the corresponding influx, which are computed through various numerical models. This study proposes an alternative formula that uses the nutrient concentration computed by a moving boundary model. The formula output was compared with measured uptake of some nutrients in different crops and soils by using experimental data extracted from the literature. The values obtained were compared with predicted uptakes by other numerical estimations as the Barber–Cushman model and our moving boundary model by using the Cushman uptake formula. Better predictions with respect to a single nutrient were obtained

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*Corresponding author. E-mail: jreginato@exa.unrc.edu.ar

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for the cases tested. Moreover, an algorithm to computing the nutrient uptake is also given.

Key Words: Nutrient uptake; Moving boundary model

ANALYSIS

Several mathematical models have been proposed to estimate the nutrient concentration at the root surface and corresponding influxes (1–4). More recently, a model has been proposed in order to compute both taking into account inter-root competition and variable root length through a moving boundary problem (5–8). The objective of this study was to compute the nutrient uptake with an alternative formula, which was a variant of the one given by Claasen and Barber (2). The results of this formula was compared with those previously expressed and proposed, in particular, by Cushman (3) for the same purpose.

The programs used in order to compute the nutrient uptake were built each time by calculating the concentration at the root surface $C(s_o,t)$, where s_o is the root radius and the corresponding net influx J. From the knowledge of these data, it was possible, by the addition of the subroutine, to compute the total uptake as time passed:

$$U_m = 2\pi s_o \sum_{i=1}^{m} \Delta l_i J_i \Delta t_i \qquad [1]$$

where U_m is the total uptake for a root element from timing zero up to time m, J_i the rate of uptake at the i's interval and Δt_i in seconds, s_o is the initial root radius and Δl_i is the root length grown. Computing the total uptake by a growing root system since it was a quantity that could be measured and used to test the theoretical model experimentally was of interest. In order to compute the total uptake by a growing root system, initially finite increments, although later infinitesimal elements, were used since the whole process was a continuous one. Since we wanted to compute the total uptake at time $t = t_{max}$, we did a partition of the interval $(0,t_{max})$ into n subintervals of length equals to $\Delta t = t_{max}/n$. From solution to the nutrient transport equations coupled with absorption kinetic, various models indicated that the rate of uptake J, changed with time for a given root element. At the beginning of the process only an amount of root equal to l_o was present and the first increment in the total uptake for the whole root system, U, was given by:

$$\Delta U_o = 2\pi s_o l_o J_o \Delta t \qquad [2]$$

where J_o is the rate of uptake of a root of zero age. The next increment for the uptake (i.e., the nutrient incorporated for the initial root volume of length l_o plus the nutrient incorporated by the growing root volume of length l_1 in the next time Δt) would be:

$$\Delta U_1 = 2\pi s_o l_o J_1 \Delta t + 2\pi s_1 \Delta l_1 J_1 \Delta t \qquad [3]$$

where J_1 was the rate of uptake of a root element one-time step old and Δl_1 was the amount of root grown at the same step (the rate of uptake and the length grown were simultaneous for each step, in contrast to the estimation of Claasen and Barber that considered Δl_1 the amount of root growth in the first step). The next increment was:

$$\Delta U_2 = 2\pi s_o l_o J_2 \Delta t + 2\pi s_o \Delta l_1 J_2 \Delta t + 2\pi s_o \Delta l_2 J_2 \Delta t \qquad [4]$$

Further steps were given by: $(n \in N, n > 1)$

$$\Delta U_n = 2\pi s_o l_o J_n \Delta t + 2\pi s_o \Delta l_1 J_n \Delta t + 2\pi s_o \Delta l_2 J_n \Delta t + \cdots + 2\pi s_o \Delta l_n J_n \Delta t \qquad [5]$$

The total uptake between time zero and time equals t_{max} was the sum of the corresponding increments for each sub-interval, that is:

$$\begin{aligned}\Delta U &= \sum_{i=1}^{n} \Delta U_i \\ &= 2\pi s_o \left[\sum_{i=1}^{n} l_o J_i \Delta t + \sum_{i=1}^{n} \Delta l_1 J_i \Delta t + \sum_{i=2}^{n} \Delta l_2 J_i \Delta t + \cdots + \sum_{i=n}^{n} \Delta l_n J_i \Delta t\right]\end{aligned} \qquad [6]$$

and, taking the limit when $\Delta t \rightarrow 0$, we deduced:

$$\begin{aligned}\Delta U &= 2\pi s_o l_o \int_0^{t_{max}} J(s)dt + 2\pi s_o \int_{l_o}^{l(t_{max})} \left[\int_t^{t_{max}} J(s)ds\right] dl(t) \\ &= 2\pi s_o l_o \int_0^{t_{max}} J(s)dt + 2\pi s_o \int_0^{t_{max}} \left[\int_t^{t_{max}} J(s)ds\right] \dot{l}(t)dt\end{aligned} \qquad [7]$$

where the first term represents the uptake for the initial root volume of length l_o, the second term represents the uptake for the successive growing volume elements and $\dot{l}(t) = dl(t)/dt$ was the root growth rate at the instant t.

The Claasen–Barber formula for the nutrient uptake was given by Claasen and Barber (2):

$$\Delta U = 2\pi s_o l_o \int_0^{t_{max}} J(s)dt + 2\pi s_o \int_0^{t_{max}} \left[\int_0^{t} J(s)ds\right] \dot{l}(t)dt \qquad [8]$$

As noted by Cushman (3), the formula [8] was incorrect and Cushman's expression to calculate the nutrient uptake was given by Cushman (3):

$$\Delta U = 2\pi s_o l_o \int_0^{t_{max}} J(s)dt + 2\pi s_o \int_0^{t_{max}} \left[\int_0^{t_{max}-t} J(s)ds \right] \dot{l}(t)dt \qquad [9]$$

In order to compare the nutrient uptake obtained through the expression of Cushman (see [9]) with our formula (see [7]), we analyzed the integrals in brackets given by:

$$C(t) = \int_0^{t_{max}-t} J(s)ds \quad \text{(Cushman)} \qquad [10]$$

$$R(t) = \int_t^{t_{max}} J(s)ds \quad \text{(Reginato–Tarzia)} \qquad [11]$$

The expression C(t) can be reformulated as:

$$\begin{aligned} C(t) &= \int_0^{t_{max}-t} J(s)ds = \int_0^{t} J(s)ds + \int_t^{t_{max}} J(s)ds + \int_{t_{max}}^{t_{max}-t} J(s)ds \\ &= \int_0^{t} J(s)ds - \int_{t_{max}-t}^{t_{max}} J(s)ds + R(t) \end{aligned} \qquad [12]$$

Then we can deduce that the sign of $C(t) - R(t)$ depends of the monotonicity of the function J, that is

$$C(t) - R(t) \begin{cases} < 0 & \text{if J is an increasing function} \\ > 0 & \text{if J is a decreasing function} \\ = 0 & \text{if J is a constant function} \end{cases} \qquad [13]$$

Thus, the estimated nutrient uptake U(t) by Cushman's formula and our formula can be compared:

$$\Delta U_{Cushman} - \Delta U_{Reginato-Tarzia} \begin{cases} < 0 & \text{if J is an increasing function} \\ > 0 & \text{if J is a decreasing function} \\ = 0 & \text{if J is a constant function} \end{cases} \qquad [14]$$

Similarly to the previous deductions, in order to assemble our expression [7] for the nutrient uptake in a computer program, we subdivided the interval of integration $(0, t_{max})$ in n time subintervals of the same amplitude $(\Delta t = t_{max}/n)$

and we propose the following algorithm:

$$\Delta U = 2\pi s_o l_o G(0) + 2\pi s_o \Delta t \sum_{i=0}^{n-1} G(t_i)\dot{L}(t_i) = 2\pi s_o l_o G(0) + 2\pi s_o \Delta t G \rightarrow (t) \bullet \dot{\vec{L}}(t) \qquad [15]$$

where • is the scalar product in $\mathscr{R}^n$ and:

$$G(0) = \Delta t \sum_{j=0}^{n-1} J(t_j) \left(= \int_0^{t_{max}} J(s)ds \right)$$

$$G(t_i) = \Delta t \sum_{j=0}^{n-i-1} J(t_{i+j}) \left(= \int_{t_i}^{t_{max}} J(s)ds \right), \quad i = 1, 2, \ldots, n-1$$

with:

$$\vec{J}(t) = \begin{pmatrix} J(0) \\ J(t_1) \\ \cdot \\ \cdot \\ J(t_{n-1}) \end{pmatrix}, \quad \vec{G}(t) = \begin{pmatrix} G(0) \\ G(t_1) \\ \cdot \\ \cdot \\ G(t_{n-1}) \end{pmatrix}, \quad \dot{\vec{L}}(t) = \begin{pmatrix} \dot{L}(0) \\ \dot{L}(t_1) \\ \cdot \\ \cdot \\ \dot{L}(t_{n-1}) \end{pmatrix}$$

where we consider $t_o = 0$ for convenience in the notation:

$$G(t_i) = G(0) - \Delta t J(0), \ldots\ldots, G(t_{n-1}) = G(0) - \Delta t \sum_{i=0}^{n-2} J(t_i),$$

$$i = 1, 2, \ldots, n-1$$

we obtain:

$$G(t_i) = \Delta t \sum_{j=i}^{n-1} J(t_j) = G(t_{i+1}) + \Delta t J(t_i), \quad i = 1, \ldots, n-1 \qquad [16]$$

The equality [16] says us that it is convenient to compute the vector $\vec{G}(t)$ beginning by the last temporal component of itself and from this we can obtain the previous temporal component and so on. The algorithm [15] and [16] can be easily translated in a program written in FORTRAN on a personal computer.

Table 1. Mg, K, and P Uptake by Pine Seedling: Observed vs. Predicted by Barber–Cushman and Moving Boundary Models

		Predicted Uptake (mmol pot^{-1})					
Crop-Soil Nutrient	Observed Uptake (mmol pot^{-1})	Barber–Cushman Model	Error %	Moving Boundary Model Using Cushman Formula [9]	Error %	Moving Boundary Model Using Formula [7]	Error %
Pine–Hapludult							
P	1.332	1.185	11.0	1.287	3.4	1.2847	3.5
K	6.663	6.285	5.7	6.536	1.9	6.5088	2.3
Mg	1.617	0.625	61.3	0.662	59.0	0.678	58.0
S (Delgado)							
Wheat (Norwood+)	0.0256	0.0047	81.7	0.0069	73.1	0.0075[a]	70.7
	0.0287	0.0297[a]	3.4	0.0255	11.3	0.0268	6.5
	0.0452	0.0592	31.0	0.0548	21.3	0.0532[a]	17.7
	0.0692	0.0935	35.1	0.0812	17.3	0.0746[a]	7.7
	0.0836	0.1294	54.8	0.1118	33.8	0.099[a]	18.4
Wheat (Norwood)	0.0109	0.0047	57.1	0.0074	31.9	0.0082[a]	24.6
	0.0234	0.0312	33.3	0.0273[a]	16.7	0.0297	26.7
	0.0452	0.0701	55.1	0.0591[a]	30.8	0.06599	46.0
	0.0561	0.106	88.9	0.0882	57.2	0.0862[a]	53.7
	0.0977	0.145	48.4	0.1220	24.9	0.1141[a]	16.8
Wheat (Mhoon+)	0.0857	0.0905[a]	5.5	0.0271	68.3	0.01247	85.5
	0.1356	0.2089[a]	54.1	0.0453	66.6	0.01383	89.8
	0.229	0.3071[a]	34.1	0.0585	74.5	0.01477	93.5
	0.2426	0.4288	76.7	0.0719[a]	70.3	0.01577	93.5
Wheat (Mhoon)	0.0555	0.0281[a]	49.4	0.0196	64.6	0.01913	65.5
	0.0836	0.0552[a]	34.0	0.0406	51.4	0.03627	56.6
	0.0764	0.0873[a]	14.3	0.0594	22.2	0.05137	32.8
	0.0836	0.1185	41.8	0.0811[a]	2.9	0.06883	17.6

[a] The value obtained represents a better prediction with respect to others.

Table 2. NO_3-N Uptake by Wheat, Rice, and Rape: Observed vs. Predicted by Analog Barber–Cushman and Moving Boundary Models

		Predicted Uptake (mmol pot^{-1})					
Nutrient Crop (Soil)	Observed Uptake (mmol pot^{-1})	Barber–Cushman Model	Error %	Moving Boundary Model Using Cushman Formula [9]	Error %	Moving Boundary Model Using Formula [7]	Error %
NO_3-N (Xuan)							
Wheat (Oxisol)	0.189	0.208[a]	10.0	0.157	16.6	0.157	16.6
Wheat (Histosol)	1.263	0.974	22.9	1.467[a]	16.2	1.468	16.2
Wheat (FluvoAquic)	2.205	1.847	16.2	2.015	8.6	2.015[a]	8.6
Rice (Oxisol)	0.514	0.640[a]	24.5	0.739	43.7	0.74	43.7
Rice (Histosol)	2.517	2.300[a]	8.62	1.657	34.1	1.658	34.1
Rape (Oxisol)	0.190	0.300	57.9	0.178	6.1	0.178[a]	6.06
Rape (FluvoAquic)	0.401	0.350	12.7	0.35	12.7	0.35[a]	12.7

[a] The value obtained represents a better prediction with respect to others.

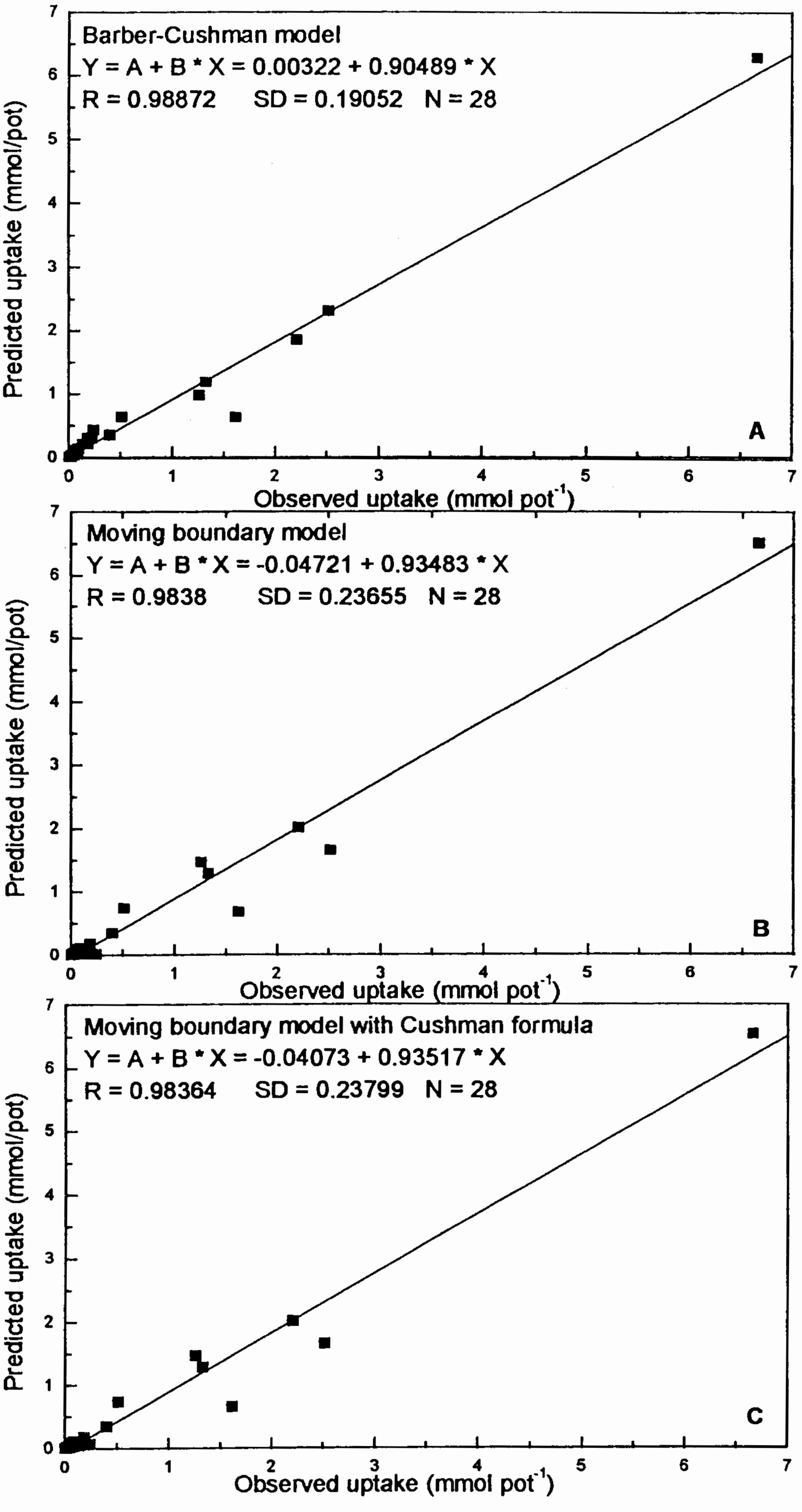

Figure 1. Comparison between the observed nutrient uptake versus the predicted nutrient uptake by: (A) Barber–Cushman model; (B) Moving boundary model, and (C) Moving boundary model using the Cushman uptake formula.

DISCUSSION

The formula [7] has been tested by using experimental data for the uptake of magnesium (Mg), potassium (K), and phosphorus (P) for loblolly pine seedlings during 180 days in a modified A horizon soil mesic Typic Hapludult (9). Moreover, the model was tested with data of sulfur (S) uptake by wheat grown on Norwood silt loam (Typic Hapludalf) and Mhoon silty clay loam (Typic Fluvaquent) for a period of 24 and 17 days, respectively, under glasshouse conditions (10).

The comparison between nutrient uptakes predicted by the preceding authors using the Barber–Cushman model through the NUTRIENT UPTAKE program (6) and the estimation of this moving boundary model (11) by using formula [7] and the Cushman uptake formula [9] is shown in Table 1.

From Table 1, we can remark, for example, that for K-uptake during 180 days in pine seedling we verify the difference that R(t) is negative (i.e., net influx is an increasing function from Eq. [14] for all time.

The model was also tested with data of NO_3 uptake by wheat, rice, and rape grown for a period of 3 to 20 days in soils Histosol (Paddy), Oxisol (Red), and Aquic Fluvents (12). This study used the Cushman equations, which were solved using a numerical integration method, a computer program written in BASIC, and executed on a personal computer. The comparison between nutrient uptakes predicted by Cushman and the estimation of this moving boundary model (11) by using formula [7] and the Cushman uptake formula [9] is shown in Table 2.

The predicted nutrient uptake by the moving boundary model by using formula [7], the predicted nutrient uptake by the moving boundary model using the Cushman uptake formula [9], and the predicted nutrient uptake by the Barber–Cushman model versus the observed nutrient uptake for all the cases tested above are compared in Fig. 1.

From graph A, the Barber–Cushman model underpredicted 0.9 times the observed uptake while the moving boundary model using formula [7] underpredicted 0.934 times the observed uptake (graph B), and the moving boundary model using the Cushman uptake formula [9] underpredicted 0.935 times the observed uptake (graph C). Thus, from graphs B and C, it is concluded that the moving boundary model predicted better results than the Barber–Cushman model because the predicted uptake was independent of the formula used for computing the nutrient uptake.

Thus, the moving boundary model using the formula [7] can be a good option in order to compute the nutrient uptake by roots.

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