

PRECISION OF MODELS OF NUTRIENT UPTAKE BY ROOTS TO LOW AND HIGH CONCENTRATIONS

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Abstract: The nutrient uptake is central in the study of the nutrition of plants. This work examines the relevance of physical models used to study the nutrient uptake by roots. We evaluate and compare physical models of nutrient uptake on fixed and moving boundaries. The moving boundary model is solved by the balance integral method. Predictions of K uptake by maize, wheat and sugar beet for low and high soil concentrations and P, K and Mg uptake by pine seedling for high concentrations are computed by both the fixed and our moving boundary model. The obtained results show that for ions with no limitations of availability, little root density variation and very low Peclet numbers all the models produce similar results. For low concentrations, great root density variation and low Peclet number the fixed model NST 3.0 model produces better predictions.

Key-words: *nutrient uptake, moving boundary model, balance integral method*

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1. INTRODUCTION

Over the past four decades different mechanistic nutrient uptake models have been developed to simulate nutrient uptake. Usually, these models consist of three basic components: (i) solute movement in the soil toward plant roots described by a continuity equation; (ii) nutrient uptake kinetics described by the Michaelis-Menten equation; (iii) nutrient uptake as a result of root growth and inter-root competition by introducing root growth and morphology parameters. Two categories of models have evolved steady state and transient models [1]. NUTRIENT UPTAKE and NST 3.0 are examples of a transient model with a numerical solution. Transient models utilizing numerical solutions are a well-established approach to mechanistic nutrient uptake models. The Barber-Cushman model is a well-known and widely-used model in this category. NUTRIENT UPTAKE model and NST 3.0 are the personal computer version of the Barber-Cushman model which is solved numerically using the Crank-Nicholson and Newton-Raphson techniques [2], [3]. In 1983 Itoh and Barber developed a submodel to the Barber-Cushman model to include nutrient uptake by root hairs. In 1986 Claassen et al. published NST 1.0 model. In 1987 Oates and Barber published NUTRIENT UPTAKE model. Both were based on the Barber-Cushman model. Later Claassen and his colleagues developed NST 2.0 and NST 3.0, which were not published in a journal. (Claassen, N. Personal communication. 2003, 24 June). NST 3.0 incorporates the Freundlich isotherm into the model so that the buffer power (b) changes as the nutrient concentration in soil solution changes [4]. From other point of view, the methodology of free and moving boundary problems have been also applied to agronomic problems. Thus, there are previous papers in this area for which the nutrient uptake have been implicitly modeled by moving boundary problems, for example [5], [6]. Explicit free and moving boundary model applied to nutrient uptake were presented in [7], [8], [9]. In this presentation, the goal is to consider an improved version of our moving boundary model applied to uptake of ions of low, medium and high availability by roots. In particular, we will study and compare the uptake of ions through model NST 3.0 and our moving boundary model (NUT BI).

2. THE MOVING BOUNDARY MODEL

This model is based on the same assumptions formulated by the Barber-Cushman model but, now, the model incorporates a new border condition for root competition which represents the net flux on the moving boundary $R(t)$. This moving boundary is given by the instantaneous half distance between root axes which is the result of the root length variation. Thus, the moving boundary is function of the instantaneous root length $\ell(t)$ which is a known function of time. A representation of the new condition

can be visualized assuming a fixed volume of soil in which the root system is distributed like a homogenous piling up by roots, i.e., we propose an idealized total root system submerged in a fixed volume of soil instead of a single root in an infinite volume of soil (Figure 1)

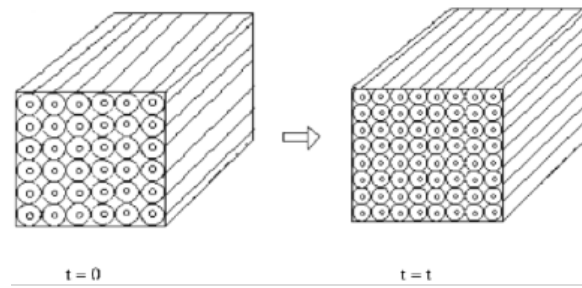


Figure 1. Homogeneous rooting in soil and his time evolution

The conditions of humidity, light and temperature are assumed to be controlled (as in a growth chamber). Biological activity is considered null. With this hypothesis, we propose the following method to compute nutrient uptake through a moving boundary model to one phase (the soil) in cylindrical coordinates:

$$\frac{\partial C}{\partial t} = D_e \frac{\partial^2 C}{\partial r^2} + \frac{D_e(1+Pe)}{r} \frac{\partial C}{\partial r}, \quad s_o < r < R(t), \quad 0 < t < T \quad (1)$$

$$C(r, 0) = C_o(r), \quad s_o < r < R(0) = R_o \quad (2)$$

$$D_e b \frac{\partial C(s_o, t)}{\partial r} + v C(s_o, t) = \frac{J_m [C(s_o, t) - C_u]}{K_m + [C(s_o, t) - C_u]}, \quad r = s_o, \quad t > 0 \quad (3)$$

$$D_e b \frac{\partial C(R(t), t)}{\partial r} + \frac{v s_o}{R(t)} C(R(t), t) = 0, \quad r = R(t), \quad 0 < t < T \quad (4)$$

with:

$$R(t) = R_o \sqrt{\frac{\ell(t)}{\ell(t)}}, \quad t > 0, \quad Pe = \frac{v_o s_o}{D_e b} \quad (5)$$

where r is the radial distance from the axis of the root [cm]; t is the time [s]; b is the buffer power [dimensionless]; D_e is the effective diffusion coefficient [$\text{cm}^2 \text{s}^{-1}$]; s_o is the root radius [cm]; v is the effective velocity of flux solution [cm s^{-1}]; R_o is the initial half distance between root axis [cm]; J_m is the maximum influx [$\text{mol cm}^{-2} \text{s}^{-1}$]; K_m is the concentration for which the influx is $J_m/2$ [mol cm^{-3}]; C_u is the threshold concentration from which influx stops [mol cm^{-3}]; $R(t)$ is the half distance between roots axis [cm]; $C_o(r)$ is the initial concentration profile in $[s_o, R_o]$ [mol cm^{-3}], $\ell(t)$ is the known root length as a function of time [cm] (the known law of root growth which can be linear, exponential or sigmoid and Pe is the Peclet number. Expression (1) is the equation of diffusive and convective transport of ions in soil and Equation (2) corresponds to the initial profile of concentrations. Equation (3) represents a null flux on the moving limit of not-transference or half mean distance between roots $R(t)$. We remark that the null flux condition imposed in this paper by equation (3) is a more realistic condition and a corrected version with respect to the similar one used in our previous model [8]. Equation (4) represents the mass balance on the root surface and the Equation (5) represents the moving boundary $R(t)$ as a function of the instantaneous root length $\ell(t)$. The solution of problem (1) – (5) is obtained by means of the integral balance method [10], [11]. Thus, we obtain the concentration and the influx on the root surface as functions of time. In order to compute the cumulative nutrient uptake by the growing root system we use a corrected version [12] of our modified version [13] of the Barber-Cushman formula [14]. This formula is given by:

$$U(t) = 2 \pi s_o \int_0^t J(t) \ell(t) dt \quad (6)$$

3. SIMULATIONS

Thus, for simulations of nutrient uptake we use two data sets extracted from literature. First, we compute the cumulative K, P and Mg uptake by pine seedling in pots during 180 days [15] using the fixed boundary models (NST 3.0) and our moving boundary model (NUT BI). The results are shown on Table 1. To compute the cumulative uptake in the two models we use the corrected formula given by (6).

Table 1. K, P and Mg uptake by pine seedlings during 180 days

Uptake (μmol)	K	P	Mg
Observed	6663	1332	1617
Predicted NTS 3	6690	1320	683
Predicted NUT-BI	6628.5	1301.3	680.7
Peclet Number	0.00057	0.0041	0.1
Root Density Variation from 0 to 180 days	0.02 a 0.20		

Second, we compute P uptake for peanut with low, intermediate and high concentrations (increasing levels of P of 0, 50, 100, 200 and 400 mg/kg of soil) during 72 days [16]. The results are shown on Table 2.

Table 2. P uptake by peanut for during 72 days

Uptake (μmol)	Peanut				
	0 P	+50 P	+100 P	+ 200 P	+ 400 P
Observed	540	640	900	1060	1320
Predicted NTS 3	708	3703	11855	32922	43841
Predicted NUT-BI	897	4680	8107	36789	43256
Peclet Number	0.022	0.027	0.025	0.029	0.03
Root Density Variation from 0 to 72 days	0.96- 3,17	1.06- 3.54	1.20- 3.97	1.08- 3.68	0.80- 2.72

Figure 2 shows the concentrations, the products $J(t) \cdot \ell(t)$ and the cumulative uptakes versus time predicted by NST 3.0 and NUT BI models for the P uptake by peanut without phosphorus addition (0 P)

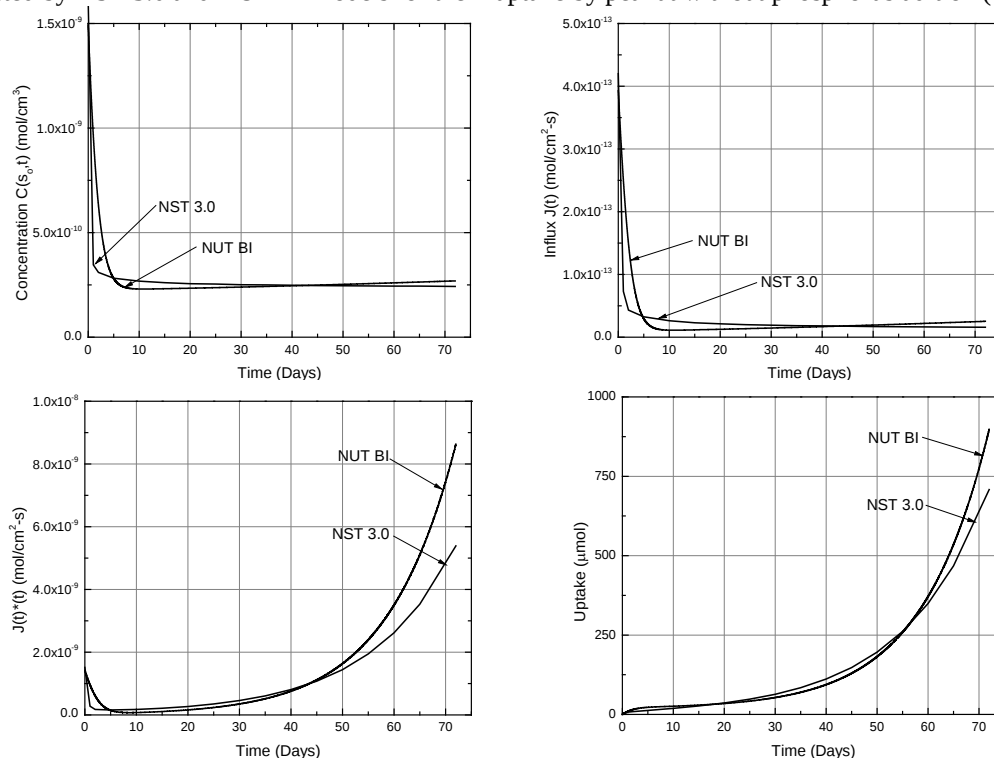


Figure 2. $C(s_o, t)$, $J(t)$, $J(t) \cdot \ell(t)$ and $U(t)$ vs. time for P uptake by peanut in the case of no addition (0 P)

4. RESULTS AND CONCLUSIONS

From Table 1, we conclude that for ions of low mobility as P and K in soils with high concentrations and very low Peclet numbers in root systems having a little root density variation the predictions are similar for the two models. From Table 2, instead, for low concentrations, low Peclet numbers and greater root density variation, better predictions are obtained by the fixed boundary model although increasing the levels of concentrations in soil, both models over predicts with great errors. Owing that the eleven input model parameters are affected by small errors, and the propagation of errors by the numerical methods used for each physical model is small, the effectiveness of the physical model used is dependent of other physical aspect as the Peclet number, root density variation and the level of concentration in soil. The precision of predictions of different models is very important with respect to the improvement of efficiency of strategies of fertilization [17] and decrease of finite set of reserves plus production of P [18].

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