Letter to the Editor

A comment on Ahmed's paper on oxygen diffusion in a sphere

A problem on diffusion of oxygen in a sphere with simultaneous absorption at a constant rate is formulated in Ahmed (1999). A popular problem of this type was studied by Crank and Gupta (1972) in which oxygen was allowed to diffuse in a body tissue in the shape of a sheet that absorbs oxygen at a constant rate. In general, these problems become nonlinear due to the presence of the free boundary and for this reason their analytical solutions are difficult to obtain. The goal of that article was to find an approximate solution applying a modification of one of the most important semi-analytical methods, the constrained integral method, proposed by Gupta and Banik (1988, 1990) in order to solve implicit free boundary problems.

In dimensionless variables, the absorption process becomes the following free boundary problem (Ahmed, 1999, (12)-(17), pp. 635):

$$u_t = u_{xx} - x, \quad s(t) < x < 1, \quad t > 0 \tag{1}$$

$$u(s(t), t) = u_x(s(t), t) = 0, \quad t > 0 \tag{2}$$

$$u_x(1, t) = 0, \quad t > 0 \tag{3}$$

$$u(x, 0) = \frac{x^3}{6}, \quad 0 \leq x \leq 1 \tag{4}$$

$$s(0) = 0. \tag{5}$$

An approximate solution of the system (1)-(5) is found assuming a polynomial of even degree in the spatial variable x for the concentration u in the region $s(t) < x < 1, t > 0$, that is:

$$u(x, t) = a(t) + b(t)\left(\frac{1-x}{1-s(t)}\right)^2 + c(t)\left(\frac{1-x}{1-s(t)}\right)^4 + d(t)\left(\frac{1-x}{1-s(t)}\right)^6 \tag{6}$$

where $a = a(t), b = b(t), c = c(t)$ and $d = d(t)$ are unknown parameters to be determined depending of the time t.

Ahmed advocates the use of the first moment of the basic equation (1) which is given by:

$$\int_{s(t)}^{1} (1-x) u_t(x, t) dx = \int_{s(t)}^{1} (1-x)(u_{xx}(x, t) - x) dx. \tag{7}$$

Applying the Leibniz rule to the left-hand side and integrating the right-hand side, we can write equation (7) as:

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$$\frac{d}{dt}\left(\int_{s(t)}^{1}(1-x)u(x,t)dx\right)+(1-s(t))u(s(t),t)\frac{ds(t)}{dt}$$
$$=(1-x)\left(u_x(x,t)-\frac{x^2}{2}\right)\Bigg|_{s(t)}^{1}+\int_{s(t)}^{1}\left(u_x(x,t)-\frac{x^2}{2}\right)dx$$

which after substitution of equations (7) and (2), and some manipulations, gives us:

$$\frac{1}{2}(1-s(t))^2\left(\frac{da(t)}{dt}+\frac{1}{2}\,\frac{db(t)}{dt}+\frac{1}{3}\,\frac{dc(t)}{dt}+\frac{1}{4}\,\frac{dd(t)}{dt}\right)$$
$$+\left(-1+\frac{1}{2}(1-s(t))\frac{ds(t)}{dt}\right)b(t)+\left(-3+\frac{2}{3}(1-s(t))\frac{ds(t)}{dt}\right)c(t)$$
$$+\left(-5+\frac{3}{4}(1-s(t))\frac{ds(t)}{dt}\right)d(t)=\frac{1}{2}\,s^2(t)(1-s(t))-\frac{1}{6}(1-s^3(t))$$

that is, the equations (21) and (23) in Ahmed's paper are wrong. Similar errors can be found in the use of the other moments.

From $u(s(t),t)=0$ and equation (6), we obtain $a(t)+b(t)+c(t)+d(t)=0$, and then the equation (25) in Ahmed's paper is also wrong because it implies $s(t)=0\forall t>0$.

For a correct resolution of the problem, we can see Gupta and Banik (1990).

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