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On the Non-Classical Heat Equation for a Semi-Space N-Dimensional

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Resumen: *We consider the non-classical heat equation in the semi-space n-dimensional domain D for which the internal energy supply depends on the heat flux on the boundary S. The problem is motivated by the modelling of temperature regulation in the medium. The solution is found for an integral representation depending on the heat flux V on S which is an additional unknown of the problem. We obtain that V must satisfy a Volterra integral equation of second kind at time t with a parameter in* $\mathbb{R}^{n-1}$. *We prove that there exists a unique local solution can be extended globally in time.*

I. Introduction, Problem and Results

In this paper we study a problem on the non-classical heat equation, in the semi-space n-dimensional space domain *D* with boundary *S*, defined by:

$$D = \mathbb{R}^+ \times \mathbb{R}^{n-1} = \left\{ (x,y) \in \mathbb{R}^n \,/\, x = x_1 > 0,\, y = (x_2, \ldots\ldots, x_n) \in \mathbb{R}^{n-1} \right\} \quad , \tag{1}$$

$$S = \partial D = \{0\} \times \mathbb{R}^{n-1} = \left\{ (x,y) \in \mathbb{R}^n \,/\, x = 0,\, y \in \mathbb{R}^{n-1} \right\} \quad . \tag{2}$$

The goal is to find the temperature *u=u(x,y,t)* such that it satisfies the following conditions:

$$\begin{cases} i)\, u_t - \Delta u = -F\left(u_x(0,y,t)\right), & x>0, \quad y \in \mathbb{R}^{n-1}, \quad t>0 \\ ii)\, u(0,y,t) = 0, & y \in \mathbb{R}^{n-1}, \quad t>0; \\ iii)\, u(x,y,0) = h(x,y), & x>0, \quad y \in \mathbb{R}^{n-1} \end{cases} \quad , \tag{3}$$

where the non-uniform source provides cooling or heating system, according to the properties of *F* with respect to the development of the heat flow at the boundary *S* [3, 4, 5]. Results for the one-dimensional case were obtained in [1, 7, 8].

Teorema

(i) The integral representation of the solution of the problem (3) is given by the following expression:

$$u(x,t) = \int_D G_1(x,y,t;\xi,\eta,0)\, h(\xi,\eta)\, d\xi\, d\eta$$
$$- \int_0^t \frac{erf\left(\frac{x}{2\sqrt{t-\tau}}\right)}{\left(2\sqrt{\pi(t-\tau)}\right)^{n-1}} \left[\int_{\mathbb{R}^{n-1}} \exp\left(-\frac{\|y-\eta\|^2}{4(t-\tau)} \right) F(V(\eta,\tau))\, d\eta \right] d\tau \tag{4}$$

where:

$$K(x,y,t;\xi,\eta,\tau) = \frac{1}{\left(2\sqrt{\pi(t-\tau)}\right)^n} \exp\left(-\frac{(x-\xi)^2 + \|y-\eta\|^2}{4(t-\tau)} \right), (x,y) \in \mathbb{R}^n, (\xi,\eta) \in \mathbb{R}^n, t > \tau, \tag{5}$$

$$G_1(x,y,t;\xi,\eta,\tau) = K(x,y,t;\xi,\eta,\tau) - K(-x,y,t;\xi,\eta,\tau), \tag{6}$$

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and the function $V = V(y,t)$*, defined by* $V(y,t) = u_x(0,y,t),\ y \in \mathbb{R}^{n-1}, t > 0$ *(heat flux on the boundary* $x = 0$*), satisfies the following Volterra integral equation of second kind:*

$$V(y,t) = \int_D G_{1x}(0,y,t;\xi,\eta,0)\,h(\xi,\eta)\,d\xi\,d\eta - 2\int_0^t \frac{1}{\left(2\sqrt{\pi(t-\tau)}\right)^n}\left[\int_{\mathbb{R}^{n-1}} \exp\left(-\frac{\|y-\eta\|^2}{4(t-\tau)}\right) F(V(\eta,\tau))\,d\eta\right] d\tau \tag{7}$$

in the variable $t > 0$ *with* $y \in \mathbb{R}^{n-1}$ *a parameter.*

(ii) Under the hypothesis

$$h \in C^0(D), \quad F \in C^0(\mathbb{R}) \text{ and locally Lipschitz in } \mathbb{R}, \tag{8}$$

there exists a unique solution of the problem (3), locally in time, which can be extended globally in times.

Proof. We use the methodology [2] in order to obtain the integral representation of the solution of the problem (3). With data (8) we can verify the hipothesis (H1)-(H3) and (H5)-(H8) in order to apply the Theorems 1.2 [6, pp. 91] and 1.3 [6, pp. 97] to the integral equation (7) which can be extended globally in times. □

Corollary

When the initial temperature h is constant and the solution of the integral equation (7) is independent of $y \in \mathbb{R}^{n-1}$*, we find the results obtained for the one-dimensional case in* [1].

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